NAME _____

Precalculus Honors Summer Math Packet

<u>DIRECTIONS/INFORMATION:</u>

- This packet contains review problems from your most recent math class and represents the types of mathematics knowledge your teacher expects you to have <u>before</u> entering Precalculus.

- The packet is divided into eight one-week sections that will allow you to develop a schedule for completing the entire packet. Follow the directions given in each section of the packet. Show your work and give full explanations where necessary. If additional space is needed, complete the work on a separate sheet of paper and attach it to the packet.

- If you have difficulty with any of the problems in the packet, refer back to your classroom notes from the school year and use the website links provided in each section of the packet.
    - After using the website information, circle the problems about which you still have questions.

Thank you in advance for completing this packet <u>by the first day of school</u>. We look forward to working with you.

I. Linear and Quadratic Equations & Inequalities

Solve each inequality and graph the solution on the number line to the right.

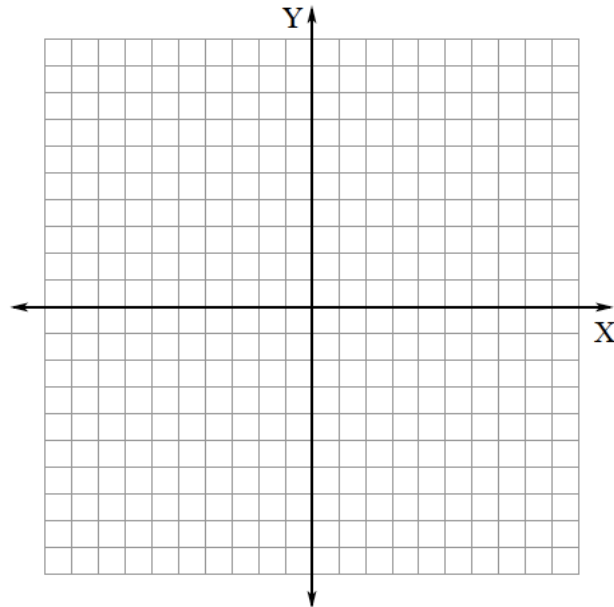1. $x+3 \geq 11 \: or \: x-3 \leq -5$

2. $-7 \leq -2x+5 \leq 1$

3. $|2x+3| < 6$

4.   Graph the linear inequality

$3x-2y \leq 12$

5.   Factor each polynomial completely. That is, write each as a product of prime polynomials.

   a. $3x^2 + 7x + 4$

   b. $3x^3 + x^2 + 12x + 4$

   c. $t^4 - 22t^2 + 40$

   d. $64x^8 - 16y^4$

   e. $x^3 - 125$
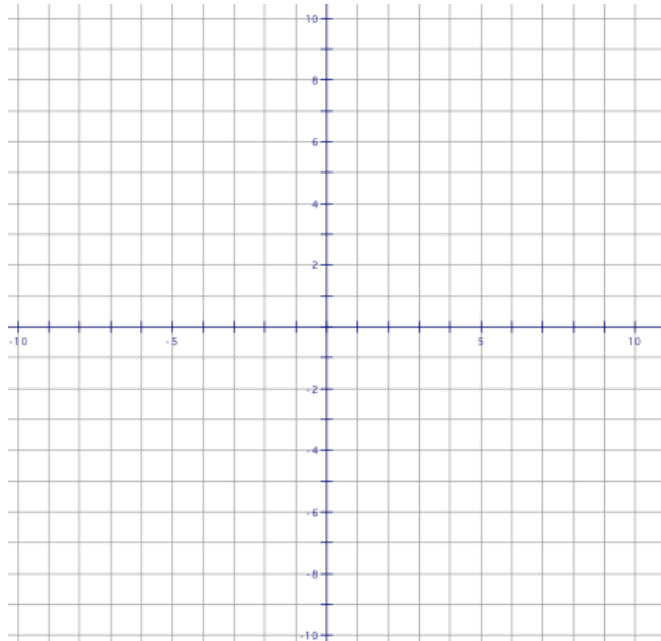
   f. $6x^3 + 48$

   g. $6x^2y - 21x^2 - 4y + 14$

6. Transform the equation $y = 3x^2 + 6x + 5$ into vertex form, $y - k = a(x-h)^2$.

    Write down the vertex _____

    Write down the y – intercept _____

    Solve for the x – intercepts and graph the parabola.

Suggested Resources: If you are having trouble with any of these problems, you may find the links below helpful.

Khan Academy:
https://www.khanacademy.org/math/algebra/linear_inequalities/compound_absolute_value_inequali/v/compound-inequalities

https://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-linear-inequalities/v/graphing-inequalities

https://www.khanacademy.org/math/algebra/quadratics/factoring_quadratics/v/factoring-quadratic-expressions

https://www.khanacademy.org/math/algebra/quadratics/completing_the_square/v/ex3-completing-the-square

II. Simplifying Rational Expressions & Solving Rational Equations

1.   Perform the indicated operation and simplify.

a. $\dfrac{x^2-3x-10}{x^2+x-2}$

b. $\dfrac{x^2-16}{9-x}\cdot\dfrac{x^2+x-90}{x^2+14x+40}$

c. $\dfrac{1}{5x^2}\div\dfrac{9x-36}{5x^3-35x^2}$

d. $\dfrac{1}{x+1}+\dfrac{x}{x-6}-\dfrac{5x-2}{x^2-5x-6}$

e. $\dfrac{\dfrac{x-2}{x+3}}{\dfrac{1}{x+\dfrac{x-1}{2}}}$

2.   Solve the following equations. Indicate any extraneous roots.

a. $\dfrac{1}{x-8} - 1 = \dfrac{7}{x-8}$

b. $\dfrac{x-2}{x+3} - 1 = \dfrac{3}{x+2}$

c. $\dfrac{x+5}{x^2+x} = \dfrac{1}{x^2+x} - \dfrac{x-6}{x+1}$

Suggested Resources: If you are having trouble with any of these problems, you may find the links below helpful.

Khan Academy:
https://www.khanacademy.org/math/algebra/rational-expressions/rational_expressions/v/adding-and-subtracting-rational-expressions

https://www.khanacademy.org/math/algebra/exponent-equations/exponent-properties-algebra/v/negative-and-positive-exponents

III. Simplifying Radical Expressions & Solving Radical Equations

1. Rewrite each in simplest radical form

    a. $\sqrt{360x^{10}y^3z^7}$

    b. $\dfrac{x\sqrt{3}}{3-\sqrt{x}}$

    b. $\dfrac{4+\sqrt{2}}{\sqrt{2}-2\sqrt{5}}$

2. Solve each equation, indicating any extraneous roots.

    a. $\sqrt{\dfrac{2x}{5}} = \sqrt{3x-58}$

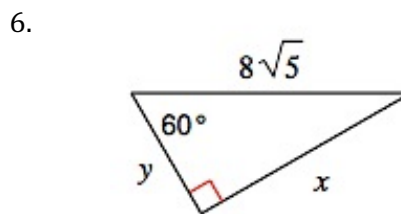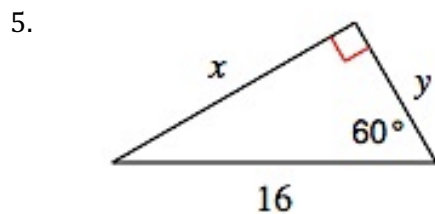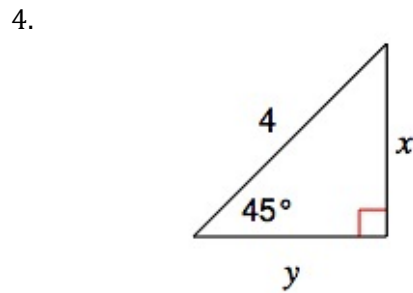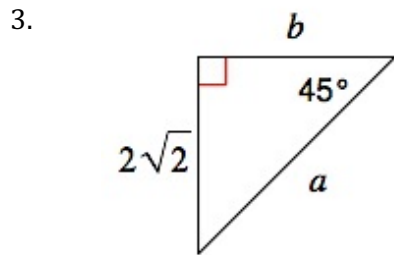   b. $3\sqrt{x-1}-11 = 2x$

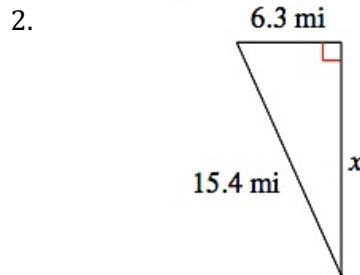   c. $\sqrt{x}-\sqrt{x-4} = 2$

Suggested Resources: If you are having trouble with any of these problems, you may find the links below helpful.

Khan Academy:
https://www.khanacademy.org/math/algebra/exponent-equations/radical_equations/v/solving-radical-equations

IV. Right Triangles

Solve for the missing sides or angles in the triangle using Pythagorean Theorem or Special Right Triangles.

1.

11.9 km

x

14.7 km

2.

6.3 mi

x

15.4 mi

3.

b

45°

$2\sqrt{2}$

a

4.

4

x

45°

y

5.

x

y

60°
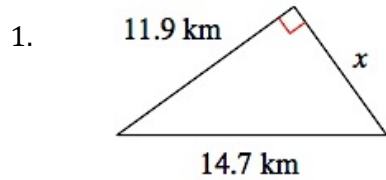
16

6.

$8\sqrt{5}$

60°

y

x

Suggested Resources: If you are having trouble with any of these problems, you may find the links below helpful.

Khan Academy:
https://www.khanacademy.org/math/geometry/right_triangles_topic/pyth_theor/v/the-pythagorean-theorem

https://www.khanacademy.org/math/geometry/right_triangles_topic/special_right_triangles/v/30-60-90-triangle-side-ratios-proof

V. Exponentials and Logarithms – complete all except (2) without a calculator

1. Solve for the given variable.

   a. $4^x = 32$

   b. $9^{-x} = \sqrt{3}$

   c. $125^{x+1} = \frac{1}{5}$

2. Transform to either a base 10 or natural logarithm and then write the value to the nearest tenth.

   a. $\log_2 5$

   b. $\log_4 9$

   c. $\log_7 10$

3. Solve for the given variable using logarithms, leave your answer in exact form.

   a. $3^x = 10$

   b. $5^{x+3} = 9$

   c. $2^{1-x} = 3^x$

4. Solve for the given variable.

   a. $y = \log_2 256$

   b. $3 = \log_4(13x - 1)$

   c. $2 = \log_b 324$

5. Rewrite as a single logarithm of a single argument and simplify

   a. $\frac{1}{2}\log_2 x + 2\log_2 y + 3\log_2 z$

   b. $2\log_3 m + 5\log_2 n + 3\log_2 mn$

6. Solve for x:

   a. $\log_6(x) + \log_6(x + 5) = 1$

   c. $\log_4(4) + \log_4(x + 2) = 2$

   b. $\log_3(10x + 2) - \log_3(x + 1) = 2$

   d. $2\log_2(x) - 3 = 1$

7. Given $\log_b 2 \approx 0.5$ and $\log_b 3 \approx 0.7$, evaluate the following:

   a. $\log_b 6$

   b. $\log_b 12$

   c. $\log_b 1.5$

Suggested Resources: If you are having trouble with any of these problems, you may find the links below helpful.

Khan Academy:
https://www.khanacademy.org/math/trigonometry/exponential_and_logarithmic_functions

PatrickJMT
http://patrickjmt.com/solving-exponential-equations/
http://patrickjmt.com/?s=logarithms

VI. Sequences and Series

Directions: answer questions #1-4 using the sequences below

    a. $5, 12, 19, 26, 33, \ldots$

                    c. $297, 99, 33, 11, \ldots$

    b. $3, 6, 12, 24, 48, \ldots$ d. $3, -6, -15, -2$

1. Determine if the sequences above are arithmetic or geometric. If arithmetic, state the common difference. If geometric, state the common ratio.

    a.   c.

    b.   d.

2. Determine an explicit equation for each of the sequences above.

    a.   c.

    b.   d.

3. Find the 10th term in each of the sequences above.

    a.   c.

    b.   d.

4. Find sum of the first 10 terms in each of the sequences above.

    a.   c.

    b.   d.

Week 6 –continued

5. A bouncy ball is dropped from the top of the Sears Tower (1,450 feet tall). Each time it strikes the ground, it bounces up to 75% of the previous height.

   a. How high will the ball bounce after it strikes the ground for the 3rd time?

   b. How high will the ball bounce after it strikes the ground for the nth time?

   c. How many times does it strike the ground before its bounce is less than 6 inches?

6. Determine the explicit equation for an arithmetic sequence with $t_2 = 10$ and $t_{29} = 91$.

7. Determine sums of the following infinite geometric series:

   a. $4 + 2 + 1 + \frac{1}{2} + \ldots$

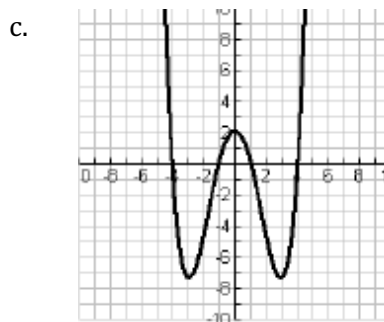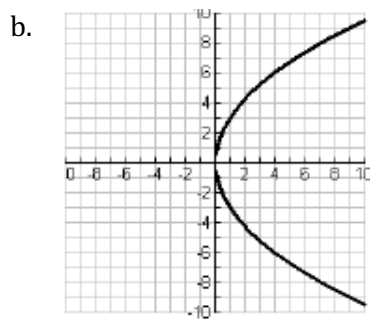   b. $12 + 4 + \frac{4}{3} + \ldots$

Suggested Resources: If you are having trouble with any of these problems, you may find the links below helpful.
Khan Academy

https://www.khanacademy.org/math/trigonometry/seq_induction/seq_and_series/v/explicit-and-recursive-definitions-of-sequences

Week 7 –

VII. Complex Numbers

1. Add or subtract:

    a. $(4 - 2i) + (3 + 6i)$ b. $(1 + 2i) - (5 + 8i)$ c. $(-3 + i) + (-4 - i)$

2. Multipy

    a. $\sqrt{-40}\sqrt{-18}$ b. $(4 - 2i)(4 + 2i)$ c. $(5 - i)(1 + 6i)$

3. Simplify the following powers of i

    a. i^{12} b. i^{107} c. i^{-21}

4. Rationalize: $\frac{2+5i}{3-4i}$

5. Determine the quadratic equation that has $3 - 2i$ as one of its roots

6. Find the solutions to the quadratic equation: $x^2 - 4x + 5 = 0$

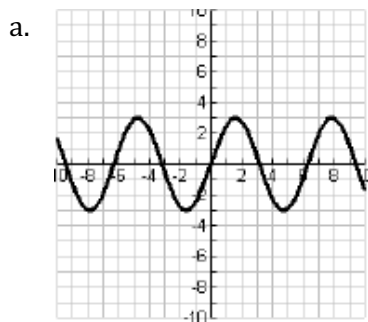7. Solve for x: $(2 + i)x + i = 10i$

Suggested Resources: If you are having trouble with any of these problems, you may find the links below helpful.

Khan Academy

https://www.khanacademy.org/math/trigonometry/imaginary_complex_precalc/complex_num_precalc/v/complex-numbers

Week 8 –

VIII. Functions

1. Determine whether each of the following are functions:

a.

b.

c.

2. Find the domain and range of the functions below:

   a. $f(x) = \frac{1}{x-4}$

   b. $g(x) = \sqrt{2x - 5}$

   c. $h(x) = 2^x + 3$

3. If $f(x) = 3x - 1$ and $g(x) = x^2$, evaluate the following:

   a. $f(g(2))$

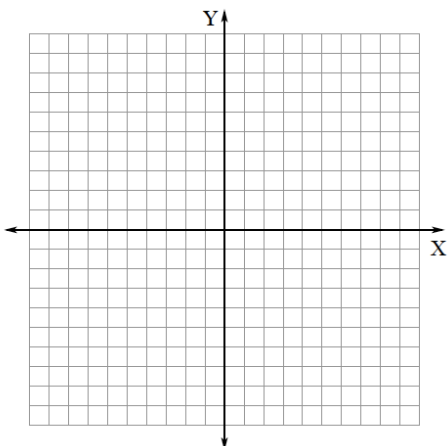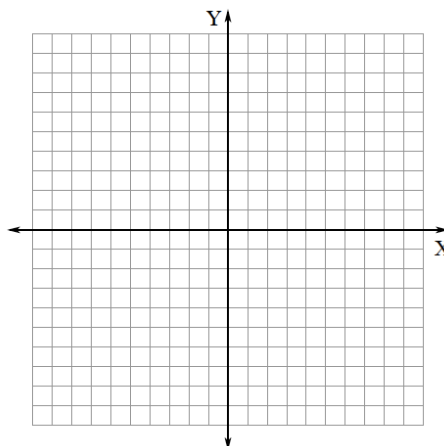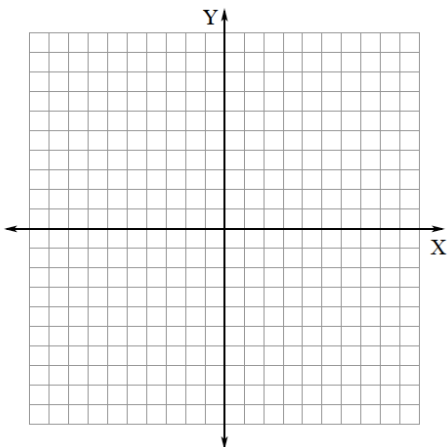   c. $f(g(x))$

   b. $f(f(5))$
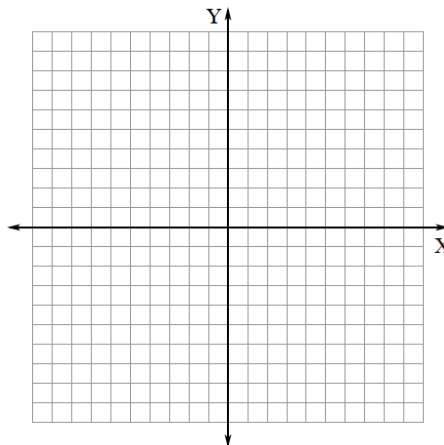
   d. $g(f(x))$

4. Sketch
   a. $f(x) = \sqrt{x-2} + 3$

   c. $f(x) = (x+1)^2 - 5$

   b. $f(x) = (x-3)^3 + 2$

   d. $f(x) = \frac{1}{x-4} + 2$

Suggested Resources: If you are having trouble with any of these problems, you may find the links below helpful.

Khan Academy:
https://www.khanacademy.org/math/trigonometry/functions_and_graphs/analyzing_functions/v/graphs-of-square-root-functions

PatrickJMT
http://patrickjmt.com/?s=domain+and+range